Решение системы линейных уравнений третьего порядка методом Крамера

Решим систему линейных уравнений, используя метод Крамера.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Знак системы |  |  | x1 | + | 2 | x2 | - | 2 | x3 | = | -8 |
| - | 3 | x1 | - | 3 | x2 | + | 3 | x3 | = | 18 |
|  | - | x1 | - | 2 | x2 | + | 3 | x3 | = | 5 |

Запишем формулы Крамера:

x1 = det A1 / det A

x2 = det A2 / det A

x3 = det A3 / det A

На ноль делить нельзя. Поэтому если det A равен нулю, то использовать формулы Крамера невозможно.

Вычислим det A.

det A состоит из коэффициентов левой части системы уравнений.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Знак системы |  |  | x1 | + | 2 | x2 | - | 2 | x3 | = | -8 |
| - | 3 | x1 | - | 3 | x2 | + | 3 | x3 | = | 18 |
|  | - | x1 | - | 2 | x2 | + | 3 | x3 | = | 5 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| det A = |  | 1 | 2 | -2 |  | = |
| -3 | -3 | 3 |
| -1 | -2 | 3 |

К элементам строки 3 прибавляем соответствующие элементы строки 1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | -2 |  |
| -3 | -3 | 3 |
| -1 + 1 | -2 + 2 | 3 + ( -2) |

Данное элементарное преобразование не изменит значение определителя.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| = |  | 1 | 2 | -2 |  | = |
| -3 | -3 | 3 |
| 0 | 0 | 1 |

Разложим определитель по элементам строки 3.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -2 |  | | -3 | -3 | 3 | | 0 | 0 | 1 | | Номер строки 3 Номер столбца 1 |  | Элемент |  |  | Строку 3 и столбец 1 вычеркнули |
| ( -1) 3 + 1 | \* | 0 | \* |  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 2 | -2 |  |  | | -3 | 3 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -2 |  | | -3 | -3 | 3 | | 0 | 0 | 1 | | Номер строки 3 Номер столбца 2 |  | Элемент |  | Строку 3 и столбец 2 вычеркнули |
| ( -1) 3 + 2 | \* | 0 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 1 | -2 |  |  | | -3 | 3 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -2 |  | | -3 | -3 | 3 | | 0 | 0 | 1 | | Номер строки 3 Номер столбца 3 |  | Элемент |  | Строку 3 и столбец 3 вычеркнули |
| ( -1) 3 + 3 | \* | 1 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 1 | 2 |  |  | | -3 | -3 | |

Произведения суммируются. Если элемент равен нулю, то и произведение тоже равно нулю.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| = ( -1) 3 + 3 \* 1 \* | | | | | |  | | 1 | 2 |  | = |
| -3 | -3 |
| = |  | 1 | 2 |  | = | |
| -3 | -3 |

= 1 \* ( -3) - 2 \* ( -3) =

= -3 + 6 =

= 3

det A не равен нулю. Использование формул Крамера возможно.

Вычислим det A1   [Скрыть](javascript:showInfo(5))

Необходимо заменить столбец 1 в det A на столбец свободных членов системы.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Система |  | det A |  | det A1 |
| |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Знак системы |  |  | x1 | + | 2 | x2 | - | 2 | x3 | = | -8 | | - | 3 | x1 | - | 3 | x2 | + | 3 | x3 | = | 18 | |  | - | x1 | - | 2 | x2 | + | 3 | x3 | = | 5 | |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -2 |  | | -3 | -3 | 3 | | -1 | -2 | 3 | |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | -8 | 2 | -2 |  | | 18 | -3 | 3 | | 5 | -2 | 3 | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| det A1 = |  | -8 | 2 | -2 |  | = |
| 18 | -3 | 3 |
| 5 | -2 | 3 |

К элементам столбца 3 прибавляем соответствующие элементы столбца 2.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | -8 | 2 | -2 + 2 |  |
| 18 | -3 | 3 + ( -3) |
| 5 | -2 | 3 + ( -2) |

Данное элементарное преобразование не изменит значение определителя.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| = |  | -8 | 2 | 0 |  | = |
| 18 | -3 | 0 |
| 5 | -2 | 1 |

Разложим определитель по элементам столбца 3.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | -8 | 2 | 0 |  | | 18 | -3 | 0 | | 5 | -2 | 1 | | Номер строки 1 Номер столбца 3 |  | Элемент |  | Строку 1 и столбец 3 вычеркнули |
| ( -1) 1 + 3 | \* | 0 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 18 | -3 |  |  | | 5 | -2 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | -8 | 2 | 0 |  | | 18 | -3 | 0 | | 5 | -2 | 1 | | Номер строки 2 Номер столбца 3 |  | Элемент |  | Строку 2 и столбец 3 вычеркнули |
| ( -1) 2 + 3 | \* | 0 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | -8 | 2 |  |  | | 5 | -2 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | -8 | 2 | 0 |  | | 18 | -3 | 0 | | 5 | -2 | 1 | | Номер строки 3 Номер столбца 3 |  | Элемент |  | Строку 3 и столбец 3 вычеркнули |
| ( -1) 3 + 3 | \* | 1 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | -8 | 2 |  |  | | 18 | -3 | |

Произведения суммируются. Если элемент равен нулю, то и произведение тоже равно нулю.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| = ( -1) 3 + 3 \* 1 \* | | | | | |  | | -8 | 2 |  | = |
| 18 | -3 |
| = |  | -8 | 2 |  | = | |
| 18 | -3 |

= -8 \* ( -3) - 2 \* 18 =

= 24 - 36 =

= -12

Вычислим det A2

Необходимо заменить столбец 2 в det A на столбец свободных членов системы.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Система |  | det A |  | det A2 |
| |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Знак системы |  |  | x1 | + | 2 | x2 | - | 2 | x3 | = | -8 | | - | 3 | x1 | - | 3 | x2 | + | 3 | x3 | = | 18 | |  | - | x1 | - | 2 | x2 | + | 3 | x3 | = | 5 | |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -2 |  | | -3 | -3 | 3 | | -1 | -2 | 3 | |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | -8 | -2 |  | | -3 | 18 | 3 | | -1 | 5 | 3 | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| det A2 = |  | 1 | -8 | -2 |  | = |
| -3 | 18 | 3 |
| -1 | 5 | 3 |

К элементам столбца 3 прибавляем соответствующие элементы столбца 1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | -8 | -2 + 1 |  |
| -3 | 18 | 3 + ( -3) |
| -1 | 5 | 3 + ( -1) |

Данное элементарное преобразование не изменит значение определителя.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| = |  | 1 | -8 | -1 |  | = |
| -3 | 18 | 0 |
| -1 | 5 | 2 |

К элементам строки 3 прибавляем соответствующие элементы строки 1, умноженные на 2.   [Скрыть](javascript:showInfo(10))

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | -8 | -1 |  |
| -3 | 18 | 0 |
| -1 + 1 \* 2 | 5 + ( -8) \* 2 | 2 + ( -1) \* 2 |

Данное элементарное преобразование не изменит значение определителя.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| = |  | 1 | -8 | -1 |  | = |
| -3 | 18 | 0 |
| 1 | -11 | 0 |

Разложим определитель по элементам столбца 3.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | -8 | -1 |  | | -3 | 18 | 0 | | 1 | -11 | 0 | | Номер строки 1 Номер столбца 3 |  | Элемент |  | Строку 1 и столбец 3 вычеркнули |
| ( -1) 1 + 3 | \* | -1 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | -3 | 18 |  |  | | 1 | -11 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | -8 | -1 |  | | -3 | 18 | 0 | | 1 | -11 | 0 | | Номер строки 2 Номер столбца 3 |  | Элемент |  | Строку 2 и столбец 3 вычеркнули |
| ( -1) 2 + 3 | \* | 0 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 1 | -8 |  |  | | 1 | -11 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | -8 | -1 |  | | -3 | 18 | 0 | | 1 | -11 | 0 | | Номер строки 3 Номер столбца 3 |  | Элемент |  | Строку 3 и столбец 3 вычеркнули |
| ( -1) 3 + 3 | \* | 0 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 1 | -8 |  |  | | -3 | 18 | |

Произведения суммируются. Если элемент равен нулю, то и произведение тоже равно нулю.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| = ( -1) 1 + 3 \* ( -1) \* | | | | | | |  | -3 | 18 |  | = |
| 1 | -11 |
| = - |  | -3 | 18 |  | = |
| 1 | -11 |

= - ( -3 \* ( -11) - 18 \* 1 ) =

= - ( 33 - 18 ) =

= -15

Вычислим det A3

Необходимо заменить столбец 3 в det A на столбец свободных членов системы.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Система |  | det A |  | det A3 |
| |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Знак системы |  |  | x1 | + | 2 | x2 | - | 2 | x3 | = | -8 | | - | 3 | x1 | - | 3 | x2 | + | 3 | x3 | = | 18 | |  | - | x1 | - | 2 | x2 | + | 3 | x3 | = | 5 | |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -2 |  | | -3 | -3 | 3 | | -1 | -2 | 3 | |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -8 |  | | -3 | -3 | 18 | | -1 | -2 | 5 | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| det A3 = |  | 1 | 2 | -8 |  | = |
| -3 | -3 | 18 |
| -1 | -2 | 5 |

К элементам строки 3 прибавляем соответствующие элементы строки 1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | -8 |  |
| -3 | -3 | 18 |
| -1 + 1 | -2 + 2 | 5 + ( -8) |

Данное элементарное преобразование не изменит значение определителя.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| = |  | 1 | 2 | -8 |  | = |
| -3 | -3 | 18 |
| 0 | 0 | -3 |

Разложим определитель по элементам строки 3.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -8 |  | | -3 | -3 | 18 | | 0 | 0 | -3 | | Номер строки 3 Номер столбца 1 |  | Элемент |  | Строку 3 и столбец 1 вычеркнули |
| ( -1) 3 + 1 | \* | 0 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 2 | -8 |  |  | | -3 | 18 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -8 |  | | -3 | -3 | 18 | | 0 | 0 | -3 | | Номер строки 3 Номер столбца 2 |  | Элемент |  | Строку 3 и столбец 2 вычеркнули |
| ( -1) 3 + 2 | \* | 0 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 1 | -8 |  |  | | -3 | 18 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 1 | 2 | -8 |  | | -3 | -3 | 18 | | 0 | 0 | -3 | | Номер строки 3 Номер столбца 3 |  | Элемент |  | Строку 3 и столбец 3 вычеркнули |
| ( -1) 3 + 3 | \* | -3 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | 1 | 2 |  |  | | -3 | -3 | |

Произведения суммируются. Если элемент равен нулю, то и произведение тоже равно нулю.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| = ( -1) 3 + 3 \* ( -3) \* | | | | | |  | 1 | | 2 |  | = |
| -3 | | -3 |
| = - 3 \* |  | 1 | 2 |  | = | | |
| -3 | -3 |

= - 3 \* ( 1 \* ( -3) - 2 \* ( -3) ) =

= - 3 \* ( -3 + 6 ) =

= -9

Ответ:

x1 = det A1 / det A = -12/3 = -4

x2 = det A2 / det A = -15/3 = -5

x3 = det A3 / det A = -9/3 = -3